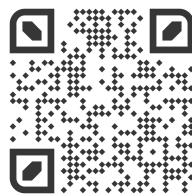


# STATISTICAL MODELING WITH APPLICATIONS 2022



# $\Sigma$ tatMod

A hybrid workshop co-organized by:

- "Gheorghe Mihoc-Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy (ISMMA), Romania
- Laboratoire de Mathématiques Raphaël Salem (LMRS), Université de Rouen - Normandie, CNRS, France
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14-15 October 2022  
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Entropy, Divergence Measures and  
Applications

Financial Mathematics

Markov and semi-Markov  
Processes

Mathematical Reliability

Renewable Energy

Statistical Methods and  
Applications

Statistics for Dependent Data

Stochastic Processes



# BOOK of ABSTRACTS

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## ABSTRACTS

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**Alexandru AGAPIE** (Bucharest Univ. of Economic Studies & ISMMA, Romania)

*Key probability results used in the theory of evolution strategies*

joint work with Luiza BĂDIN (Bucharest Univ. of Economic Studies & ISMMA, Romania), Ovidiu SOLOMON and Marius GIUCLEA (Bucharest Univ. of Economic Studies & Institute of Solid Mechanics, Romania)

*Abstract:* We present some of the most interesting probabilistic results used in a recent developed theory of a class of continuous optimization algorithms, the so-called evolution strategies.

**Ada ALBU** (INCE, Romania)

*A stochastic model for the analysis of the public debt*

*Abstract:* Theories regarding the dynamics of public debt usually refer to the optimal values for public debt, where debt should not exceed a critical threshold, beyond which it can impede economic growth. The stochastic analysis is based on a framework which includes several variables which influence the evolution of public debt in order to establish a optimal debt threshold. Sustainability concerns refer to the trajectory of public debt, using interest rates or the primary balance as variables which influence the evolution of debt. Thus, using stochastic analysis it is possible to take into consideration several debt paths, which allow an in-depth simulation of public debt dynamics. We will analyze a stochastic model for Debt Sustainability, based on VAR models which is built on variables such as short-term and long-term interest rates, the real GDP increase and the real increase of GDP deflator.

**Jean Vasile ANDREI** (Faculty of Economic Sciences, Petroleum-Gas Univ. of Ploiești, Romania)

*An empirical analysis of factors influencing buying electric and hybrid cars in European Union: assessing the buyers' willingness*

joint work with Silviu STANCIU (Dunărea de Jos Univ. of Galați, Romania), Violeta SIMA (Faculty of Economic Sciences, Petroleum-Gas University of Ploiești), Ileana Georgiana GHEORGHE (Faculty of Economic Sciences, Petroleum-Gas Univ. of Ploiești, Romania)

*Abstract:* The recent transformation of the mobility paradigm has imposed electric and hybrid as possible instruments in reducing the dependence of fossil fuels consumption and

lowering the environmental pollution. This research aimed to identify and analyze the determining and influencing factors of buying electric and hybrid cars in European Union, assessing the buyers' willingness by adopting across-sectional research model based on five research hypotheses. The proposed econometrical model allows finding the purchasing power optimal for electric and hybrid cars distribution that maximizes and confirms the factors' statistical importance of loading infrastructure and government subsidies. The results confirms a perceived relative advantage and compatibility associated with the attitude of buying electric and hybrid have a positive effect on the countries with highest values of GDP, education and degree of digitalization. Under the model assumptions, assessing the buyers' willingness for buying electric and hybrid cars in European Union is tighten determined by the technological developments in the electric vehicles industry so that they become environmentally friendly compared to fossil engine based cars.

**Alexandra ANDRICIUC** (Univ. of Bucharest & IMAR, Romania)

*Stagnation analysis for the particle swarm optimization algorithm*

joint work with Ionel POPESCU and Iulian CÎMPEAN (Univ. of Bucharest & "Simion Stoilow" Institute of Mathematics of the Romanian Academy, Romania)

*Abstract:* The PSO algorithm is a very popular one among the practitioners in ML thanks to the elegant coding manner and its interesting parallel with animal behavior. However, the mathematical literature is still lacking a solid background in this regard; for instance, one cannot say much about the convergence of the algorithm, given a precise choice for the hyperparameters. Our purpose is to take advantage of the stochastic intervention from this algorithm - not approximating it with constants as in most previous algebraic works - in order to obtain qualitative results. Hence, probabilistic instruments such as generating functions and Borel-Cantelli lemma are put to use, aiming to create a context that unifies the rigorous conditions with the go-to values from practice.

**Arefe ASADI** (Univ. of Technology of Troyes, France)

*Depth function for degradation modelling*

joint work with Mitra FOULADIRAD (École Centrale Marseille, France)

*Abstract:* Model selection is an important issue for degradation modelling, prognostic and maintenance planning. Since information on the deteriorated system is not available continuously and measurements or inspections are not necessarily periodic the classical model selection and validation in the literature cannot be used directly. In this paper, we propose a first attempt for degradation model selection with convex hull depth function. After the introduction of the problem and model, a numerical study illustrates the interest

of the proposed methodology.

**Chafiàa AYHAR** (Univ. Dr. Moulay Tahar of Saïda, Algeria)

*Nonparametric estimation of some important reliability indicators for semi-Markov processes*

joint work with Vlad Stefan BARBU (LMRS, France), Fatiha MOKHTARI and Saâdia RAHMANI (Univ. Dr. Moulay Tahar of Saïda, Algeria)

*Abstract:* We consider a semi Markov process, with a finite state space. We use the kernel method to introduce nonparametric estimators of some indicators of technical systems, such as the mean up time (MUT), the mean down time (MDT) of an arbitrary finite semi-Markov. Strong consistency and weak convergence is proved for the estimators of MUT, and MDT, as the time interval of observation becomes large.

#### *References*

- [1] C. Ayhar, V.S. Barbu, F. Mokhtaria, S. Rahmani. (2022). On the asymptotic properties of some kernel estimators for continuous-time semi-Markov processes. *Journal of Nonparametric Statistics*, 34(1), 1-21.
- [2] N. Limnios, and B. Ouhbi (2006). Nonparametric estimation of some important indicators in reliability for semi-Markov processes. *Stat Methodol*, 3(4), 341-350.
- [3] R. Pyke. (1961). Markov renewal processes with finitely many states. *Ann. Math. Statist*, 32, 1243-1259.
- [4] R. Pyke and R. Schaufele. (1964). Limit theorems for Markov renewal process. *Ann. Math. Statist*, 32, 1746-1764.
- [5] R. Pyke and R. Schaufele. (1966). The existence and uniqueness of stationary measures for Markov renewal processes. *Ann. Math. Statist*, 37, 1439-1462.
- [6] A. Sadek, N. Limnios, (2002). Asymptotic properties for maximum likelihood estimators for reliability and failure rates of Markov chain. *Comm. Statist. Theory Methods*, 31(10), 1837-1861.

**Luiza BĂDIN** (Bucharest Univ. of Economic Studies & ISMMA, Romania)

*Performance assessment of the Italian healthcare system using conditional nonparametric efficiency models*

joint work with Camilla MASTROMARCO (Univ. of Calabria, Italy) and Raffaele LAGRAVINESE (Univ. of Bari Aldo Moro, Italy)

*Abstract:* Conditional frontier models, including full and partial, robust frontiers, have evolved into an indispensable tool for exploring the impact of exogenous factors on the performance of the Decision Making Units in a fully nonparametric setup. The conditional

nonparametric framework enables the handling of heterogeneity in a formal way, allowing to explain the differences in the efficiency levels achieved by units operating under different external and/or environmental conditions. In this paper we show how this nonparametric dynamic framework is important to evaluate efficiency in the healthcare sector. We provide numerical illustrations on a dataset from the Italian healthcare system, including summaries of practical implementation details.

**Costel BĂLCĂU** (Univ. of Pitești, Romania)

*Interval weighted cumulative residual entropy for generalized Pareto distributions*

joint work with Doru CONSTANTIN (Univ. of Pitești, Romania)

*Abstract:* The concept of double truncated (interval) weighted cumulative residual entropy (IWCRE) was introduced by Yasaei Sekeh et al. (2015). In this note we compute the IWCRE for the Pareto Type I, II distributions with some power-type weight functions. The obtained results involve some special integral functions, like as the dilogarithm function and the generalized incomplete Beta function.

**Ionut BEBU** (George Washington Univ., USA)

*Personalized screening schedules for a chronic disease with application to diabetic nephropathy*

*Abstract:* Periodic evaluations are required in the clinical management of chronic diseases in order to identify opportunities for early treatment that can avert further progression and complications. For example, in nephropathy, patients with type 1 diabetes are screened annually for the onset of microalbuminuria, an early sign of chronic kidney disease. However, as our understanding of the etiology of diseases improves, there is the opportunity to design personalized schedules for future visits based on the estimated risk of progression, instead of a fixed schedule for the entire population. Herein we describe a framework for constructing such personalized screening schedules. We first fit a repeated interval proportional rate model, analogous to successive Cox PH models, with covariate effects on the risk of progression of the disease during the interval starting from the time of the current visit and up to that of the next visit. A screening interval to the next visit can then be selected to optimize some function such as limiting the probability that the event will occur (e.g.,  $< 0.05$ ) prior to the next visit, or limiting the time that progression might go undetected before the next visit. Several approaches for model selection are also described. The methods are illustrated using screening for microalbuminuria in the DCCT/EDIC study.

**Lucian BEZNEA** (Univ. of Bucharest & "Simion Stoilow" Institute of Mathematics of the Romanian Academy, Romania)

*Multiple-fragmentation stochastic processes driven by a spatial flow*

joint work with Ioan R. Ionescu (Univ. Sorbonne-Paris-Cité, France) and Oana LUPAȘCU STAMATE (ISMMA, Bucharest)

*Abstract:* We study stochastic multiple-fragmentation processes driven by a spatial flow. The final goal is actually to make a numerical simulation of the time evolution of a system of particles located on an Euclidean surface.

**Luigi-Ionuț CATANĂ** (Univ. of Bucharest, Romania)

*Application of order statistics in the study of earthquakes*

joint work with Vasile PREDA (ISMMA & Nat. Inst. for Ec. Res. INCE & Univ. of Bucharest, Romania)

*Abstract:* In this presentation we analyze the properties of Jones-Larsen multivariate distribution that describe order statistics. We also present an empirical application in the study of earthquakes.

**Corina CIPU** (Department of Applied Mathematics & CiTi, Faculty of Applied Sciences, Univ. Politehnica of Bucharest, Romania)

*Hybrid statistical models using evolutive algorithms or machine learning for demographic processes*

joint work with Emil SIMION (Department of Mathematical Methods and Models & CiTi, Faculty of Applied Sciences, Univ. Politehnica of Bucharest, Romania)

*Abstract:* In the beginning of the paper a review of the types of classical demographic models is made. Beginning with lifetables, schedule models, parametric approaches with log-linear formula provided to fit mortality curves were remembered. Indirect estimation models used to assess quality of the reported data and to derive different indicators in geographic areas with incomplete or inaccurate data and the number and type expanding of demographic variables led to projection methods and studies on changing patterns. Also, as part of statistical analysis, decomposition methods and inductive analyses were used to breaks the demographic outcomes into components. Nowadays simulation methods with stochastic processes were used. In the cross-disciplinary methodologies category, Evolutionary algorithms such as Variable-Chromosome-Length Genetic Algorithm (VCL\_GA) are useful in analyzing the demographic data when the length of the optimal demography is unknown. To improve the quality of previous estimations different machine learning techniques could be added and robust hybrid statistical models



being derived.

**Daniel CIUIU** (Technical Univ. of Civil Engineering of Bucharest & INCE, Romania)

*Monte Carlo methods to solve linear programming problems and applications to engineering: MADM in linear programming*

joint work with Cristian ANGHEL and Cornel ILINCA (Technical Univ. of Civil Engineering of Bucharest, Romania)

*Abstract:* In this paper we use the fundamental theorem of duality for linear programming in order to solve the primal and the dual problem for a constraints' problem/ menu problem. We use the fact that for any feasible solution of the constraints' problem (the problem of maximum benefit) the value of the objective function is less than the common optimal value, while for any feasible solution of the menu problem (the minimum cost problem) the value of the objective function is greater than the common optimal value. Therefore we use the bisection method to find the common optimal value: for all the current values chosen by bisection we start simulating (Monte Carlo methods) pairs of  $x$  and  $y$  such that the corresponding objective functions are equal to current value. For a given current value we stop the simulation when  $x$  is a feasible solution for the maximum problem, or when  $y$  is a feasible solution for the minimum problem. If the current value is less than the real common optimal solution, there is not a feasible solution for minimum problem having the objective function equal to the current value, and we stop with  $x$  feasible solution. Analogously, if the current value is greater than the common optimal value we stop with  $y$  feasible solution. The simulations of  $x$  and  $y$  are reduced to simulation of uniform random variables between zero and the current variable limit. The choice of Francis turbines is according to energy and cost. The optimization of the two mentioned sizes is done depending on the installed flow and the diameter of the pipe. Because there are two objective functions in this case, a MADM approach is necessary for the above Francis turbines.

**Iulian CÎMPEAN** (Univ. of Bucharest & "Simion Stoilow" Institute of Mathematics of the Romanian Academy, Romania)

*Ergodicity of Markov semigroups and application to singular SDEs on Hilbert spaces*

*Abstract:* The common principle of proving convergence to equilibrium for a Markov dynamics is to show that the corresponding semigroup exhibits some regularity and some tightness properties, and in the first part of the talk we shall recall such strategies. Then, following [1] we shall explain that the problem of mere existence of an invariant distribution can be tackled from a purely measure theoretic perspective. More precisely, we shall characterize those finite measures  $m$  for which there exists a density  $\rho$  such that  $\rho \cdot m$  is an invariant distribution for the given semigroup. The above mentioned

characterization allows to prove ergodicity of a class of singular SDEs on Hilbert spaces, as those considered in [2]

### *References*

- [1] L.Beznea, I. Cîmpean, M. Röckner. (2019). A new approach to the existence of invariant measures for Markovian semigroups. *Ann. Inst. H. Poincare Probab. Statist.*, 55(2), 977-1000. arXiv :1508.06863v3
- [2] L.Beznea, I. Cîmpean, M. Röckner. (2020). A natural extension of Markov processes and applications to singular SDEs. *Ann. Inst. H. Poincare Probab. Statist.*, 56(4), 2480-2506. arXiv :1904.01607v3

**Simona COJOCEA** (Univ. of Bucharest, Romania)

*A new Central Limit Theorem using Kolmogorov means for a particular class of random variables*

*Abstract:* In his paper from 1930 “Sur la notion de la moyenne” Kolmogorov introduces a generalized mean which was compatible with the arithmetic mean, the geometric mean and the harmonic mean. This was only the starting point of further generalization, so many new classes of generalized means have emerged. In this paper, we take a look at Kolmogorov’s original paper with a fresh perspective and we present a central limit theorem using the Kolmogorov expected value for a particular class of random variables.

**Vlad Raul CONSTANTINESCU** (Univ. of Bucharest & ISMMA, Romania)

*Global minima of overparametrized neural networks*

joint work with Ionel POPESCU (Univ. of Bucharest & "Simion Stoilow" Institute of Mathematics of the Romanian Academy, Romania)

*Abstract:* We study the geometry of global minima of the loss landscape of overparameterized neural networks. In most optimization problems, the loss function is convex, in which case we only have a global minima, or nonconvex, with a discrete number of global minima. We prove that for a family of activation functions, the locus of global minima of the loss landscape of an overparametrized neural network is a submanifold of  $\mathbb{R}^n$ . More precisely, if a neural net has  $n$  parameters and is trained on  $d$  data points, where  $n > d$ , then the locus  $M$  of global minima is an  $n - d$  dimensional submanifold of  $\mathbb{R}^n$ . Also, we give a description of the Hessian evaluated at these global minima.

**Radu CRAIU** (Univ. of Toronto, Canada)

*General behaviour of p-values under the null and alternative*

joint work with Yanbo TANG (Imperial College, UK) and Lei SUN (Univ. of Toronto, Canada)

*Abstract:* Hypothesis testing results often rely on simple, yet important assumptions about the behavior of the distribution of p-values under the null and alternative. We examine tests for one dimensional parameters of interest that converge to a normal distribution, possibly in the presence of many nuisance parameters, and characterize the distribution of the p-values using techniques from the higher order asymptotics literature. We show that commonly held beliefs regarding the distribution of p-values are misleading when the variance or location of the test statistic is not well-calibrated or when the higher order cumulants of the test statistic are not negligible. We further examine the impact of having these misleading p-values on reproducibility of scientific studies, with some examples focused on GWAS studies. Corrected tests are proposed and are shown to perform better than their traditional counterparts in various settings.

**Guglielmo D'AMICO** (Univ. "G. d'Annunzio" of Chieti-Pescara, Italy)

*ROCOF of higher order for semi-Markov processes*

joint work with Filippo PETRONI (Marche Polytechnic Univ. of Ancona, Italy)

*Abstract:* In this paper, we study the rate of occurrence of failures (ROCOF) of higher order for continuous time semi-Markov processes (SMP). This indicator gives information on whether there are a lot of failures or only a few within a time interval. It also considers the relative positioning of tuples of failures in time. Here, we consider SMP with a mixed probability distribution for the initial law of the system taking into account the possible random starting from any state with any duration. Furthermore, under suitable assumptions on the transition rates, we determine an explicit formula for the ROCOF of higher order and we recover as particular cases previous results obtained in the literature. A numerical example demonstrates the possibility of using this index in real applications.

**Ion GHIZDEANU** (Centre for Demographic Research "Vladimir Trebici", Romania)

*The change of the population forecasting models and methodologies - a requirement to increase the circulation of contingents*

joint work with Sorin AVRAM, Alexandru SIN SCHNEIDER (Centre for Demographic Research "Vladimir Trebici" & School of Advanced Studies of the Romanian Academy, Romania) and Costin Adrian CACE (Centre for Demographic Research "Vladimir Trebici", Romania)

*Abstract:* Globalization and liberalization of regional and zonal economic markets continue to rely on high population flows directed to developed economic centers. Conflicts and economic crisis accentuate international circulation beyond any political or natural restrictive measures. The single European market is perhaps the most expressive example of regional flows, which profoundly changed the number and structure of the resident population in almost all EU member states, obviously in favor of countries with an attractive job offer. Even if the intensity of global and regional flows will experience a flattening tendency as the effects of development expand spatially, they will remain decisive in the long term in the forecast of population evolution. The factors that determine the demographic flows will have to become apriority in the population forecast, especially the resident population, instead of the classical criteria based on local natural increases. The intensification and maintenance of long-term international circulation resides in economic gaps. Economic growth and progress are not yet converging. The study will present the evolution of real convergence in the EU as an argument for the prioritization of the proposed new approach to forecasting. The flows that change the demographic situation are predominantly for the 15-64-year-old population, aiming at well-paid jobs. As a result, the level of development that will be reached at some point, the level of income and labor productivity will be the criteria that will determine the resident population requirement at the end of a time horizon. The reality shows that in countries like Romania, with significant decreases in natural increases, the resident population requirement will not be able to be covered without population inflows. The proposal, which will be detailed in the study, describes the method of forecasting the population, based on the employment requirement and possible flows, including by extending it to the entire population, based on the new values of the demographic indicators, influenced by the immigrant contingents. By comparing with the forecast based on the classic demographic model, it can be concluded whether a public policy to stimulate immigration can be configured.

**Iulia Elena HIRICĂ** (Univ. of Bucharest, Romania)

*Applications of Lie symmetries associated to nonlinear Fokker-Planck equation based on different weighted entropies*

joint work with Gabriel Teodor PRIPOAE, Cristina Liliana PRIPOAE (Univ. of Bucharest, Romania) and Vasile PREDA (ISMMA & Nat. Inst. for Ec. Res. INCE & Univ. of Bucharest, Romania)

*Abstract:* The nonlinear Fokker-Planck equation (NFPE) is one of the fundamental equations in Statistical Mechanics. It governs phenomena which may be modeled by the time evolution of the probability density function of the velocity of a particle, moving under the influence of both deterministic forces and random forces. The main goal is the study of Lie symmetries of the nonlinear Fokker-Planck equation in one dimension, associated to the certain weighted entropies (Sharma-Taneja-Mittal entropy, Tsallis

entropy, Kaniadakis entropy). In some particular important cases, we found some sub-algebras, identifying their isomorphism classes in the Bianchi classification. We prove that the associated Lyapunov function is non-negative and the Bregman divergence may be interpreted as a distance function, in terms of differences between values of the Lyapunov function. We also derive the solution for the maximum entropy problem associated to the weighted entropies and a weighted generalization for some thermodynamic relations.

**Iuliana-Florentina IATAN** (Technical Univ. of Civil Engineering of Bucharest, Romania)

*Statistical modeling using Petri Networks*

joint work with Ion MIERLUȘ-MAZILU, Romică TRANDAFIR (Technical Univ. of Civil Engineering of Bucharest, Romania) and Mihăiță DRĂGAN (Univ. of Bucharest, Romania)

*Abstract:* The purpose of this paper is to develop a methodology for modeling a practical problem with the help of Petri nets. Petri nets represent a special category of directed graphs, that differs from a directed graph by the fact that the set of nodes of the graph consists of two distinct sets P and T. The evolution of the Petri net is defined by the movement of the tokens; starting from the initial state, tokens pass from one place to another, by executing the transition. Visual Simnet is a software specialized in Petri net programming and simulation of the model, that it represents. It is suitable for statistical processes modeled with networks Petri. In our example, a race is started between two cars A and B. Then when the starter receives the "ready" signal from all the machines, it gives the start signal and the cars start the race.

**Gheorghe Dan ISBASOIU** (Faculty of Economic Sciences, Petroleum-Gas Univ. of Ploiești, Romania)

*Methods of choosing renewable energy classes*

joint work with Irina Gabriela RADULESCU (Petroleum-Gas Univ. of Ploiești, Faculty of Economic Sciences, Romania), Andrei Marcel MANOLE (Valahia University, Doctoral School Economics, Romania), Mihai MIEILA (Valahia Univ., Faculty of Economic Sciences, Romania)

*Abstract:* The starting point in this study is that not all categories of renewable energies can be used in certain areas of the Earth. It is thus necessary to refer to a certain area, however limited, but which has the same basic characteristics for the production of these types of energies. The area referred to has, from the point of view of natural factors, the ability to produce energy only on the basis of solar, wind and the internal temperature of

the earth. This article identifies ways to determine the level of energy and water consumption related to individual households in a homogeneous area in terms of available natural resources. Energy consumption contains the consumption of electricity as well as the energy that is consumed through the consumption of natural gas. The results were obtained following the application of a survey in the defined area and respectively the application of the regression and dispersion analysis models related to the internal factors at the household level, those that were considered as determinants in the level of consumption (occupied surfaces: land, constructions, respectively number of persons in the household, employed population). The overall objective was to define the type of renewable energy that can be used, in relation to household resources, as well as to identify the needs for technological change. We refer in this situation to the variable sizing of wind turbines, for example, or to the type of energy obtained through cogeneration, depending on the household. As a derivative objective achieved is the planning of the implementation of these solutions at the level of the entire area, considered homogeneous.

**Dorin JULA** (IPE, Ecological Univ. of Bucharest, Romania)

*Population dynamics and the evolution of energy consumption. Cliometric analysis*

joint work with Diana Mihaela JULA (School of Advanced Studies of the Romanian Academy, Romania)

**Emmanouil-Nektarios KALLIGERIS** (LMRS, France)

*Dynamic stochastic modeling of time-series incidence data*

joint work with Alex KARAGRIGORIOU (Univ. of the Aegean, Greece) and Christina PARPOULA (Panteion Univ. of Social and Political Sciences, Greece)

*Abstract:* In this work, a Markov Regime Switching Model of Conditional Mean with covariates is proposed and investigated for the analysis of incidence data. The components of the model are selected by both penalized likelihood techniques in conjunction with the Expectation Maximization algorithm, with the goal of achieving a high level of robustness regarding the modeling of dynamic behaviors of epidemiological data. In addition to statistical inference, Changepoint Detection Analysis is performed for the selection of the number of regimes, which reduces the complexity associated with Likelihood Ratio Tests. Within this framework, a three-phase procedure for modeling incidence data is proposed and tested via real and simulated data.

**Emmanuel LEPINETTE** (Paris-Dauphine Univ., France)

*Robust discrete-time super-hedging strategies under AIP condition and under price uncertainty*

joint work with Meriem EL MANSOUR (Paris-Dauphine Univ., France)

*Abstract:* We solve the problem of super-hedging European or Asian options for discrete-time financial market models where executable prices are uncertain. The risky asset prices are not described by single-valued processes but measurable selections of random sets that allows to consider a large variety of models including bid-ask models with order books, but also models with a delay in the execution of the orders. We provide a numerical procedure to compute the minimum price under a weak no-arbitrage condition, the so-called AIP condition, under which the prices of the non negative European options are non negative. This condition is weaker than the existence of a risk-neutral martingale measure but it is sufficient to numerically solve the super-hedging problem. We illustrate our method by a numerical example.

**Maria LONGOBARDI** (Univ. degli Studi di Napoli FEDERICO II, Italy)

*Entropy: a useful unified formulation*

joint work with Narayanaswamy BALAKRISHNAN (McMaster Univ., Canada) and Francesco BUONO (Univ. degli Studi di Napoli FEDERICO II, Italy)

*Abstract:* A general formulation of entropy is proposed. It depends on two parameters and includes Shannon, Tsallis and fractional entropy, all as special cases. This measure of information is referred to as fractional Tsallis entropy and some of its properties are then studied. In order to introduce the corresponding entropy in the context of Dempster-Shafer theory of evidence, the definition and some properties of the fractional Deng entropy are given.

**Andreas MAKRIDES** (Univ. of Cyprus & Univ. of the Aegean, Greece)

*A financial view of semi-Markov modeling*

joint work with Christos MESELIDIS and Alex KARAGRIGORIOU (Univ. of the Aegean, Greece)

*Abstract:* An innovative approach of loss ratio forecasting is developed using a special type of semi-Markov processes. Three levels of loss ratio are considered as the states of a semi-Markov process, and semi-Markov process methodology is employed for estimating transition probabilities of loss ratio levels transit from a predefined level to another one.

**Ioannis MAVROGIANNIS** (LMRS, France)

*dsmmR: Estimation and Simulation of Drifting Semi-Markov Models*

joint work with Vlad Stefan BARBU and Nicolas VERGNE (LMRS, France)

*Abstract:* Drifting Semi-Markov models (DSMM) are inhomogeneous Markov models which capture the heterogeneities of sequences with more flexibility than Semi-Markov models, a Markov model generalization. This is achieved by describing the Drifting Semi-Markov kernel as a linear combination of  $d + 1$  Semi-Markov kernels for every instance of a sequence. In this presentation, we will introduce and define three different Drifting Semi-Markov model specifications, show how the estimation of these models is possible and showcase through the R package dsmmR how we can estimate, define and use parametric and non-parametric drifting semi-Markov models to simulate sequences.

**Fatiha MOKHTARI** (Univ. Dr. Moulay Tahar of Saïda, Algeria)

*Limit laws and strong consistency of non parametric estimators for the main characteristics of continuous-time finite space semi Markov processes*

joint work with Chafiàa AYHAR, Saâdia RAHMANI (Univ. Dr. Moulay Tahar of Saïda, Algeria), Vlad Stefan BARBU (LMRS, France)

*Abstract:* The present work is devoted to introduce the kernel estimators of the semi-Markov kernel and of the sojourn time distribution (conditional and unconditional), as well as the estimators of Radon-Nikodym derivatives of the previous measures. Asymptotic properties of the main characteristics of these estimators are established under standard structural and regularity assumptions. This method is illustrated by considering a three state semi-Markov process example with numerical evaluations.

**Marian NECULA** (Bucharest Univ. of Economic Studies, Romania)

*Current advances in spatial thresholding techniques with applications in remote sensing data*

*Abstract:* Thresholding (binary classification) is an important technique used for image segmentation/classification. Present study reviews up-to-date advances in spatial thresholding techniques with applications in vegetation estimation using remote sensing data. We provide performance analysis of the algorithms using a consensus-based approach between a half-dozen performance metrics compared with results from a manual-based threshold selection, applied to a sample of remote sensing data for Romanian cities.



**Adriana NISTOR** (Univ. of Bucharest, Romania)

*Risk measures*

joint work with Ionel POPESCU (Univ. of Bucharest & IMAR, Romania)

*Abstract:* The risk measures are statistical instruments used in predicting the risk and volatility of a portfolio based on historical data. The most popular risk measures are the standard deviation, Sharpe ratio, beta, Value at Risk (VaR), and Conditional Value at Risk (CVaR). What all these have in common is that they are measuring the risk (random variable) with a real number. The downside is that all these classical measures are unable to capture (or predict) the change in the risk regime. Our proposal is a risk measure that is associating to the random variable not a single real value, but a step function. The aim is to capture the magnitude of the risk under different risk regimes. In this presentation, we aim at covering a few theoretical aspects of the proposed risk measure.

**Mihaela-Georgiana OPREA** (Centre for Demographic Research "Vladimir Trebici" & School of Advanced Studies of the Romanian Academy, Romania)

*Main features of the migration phenomenon among women*

joint work with Carmen GHEORGHE (Centre for Demographic Research "Vladimir Trebici" & School of Advanced Studies of the Romanian Academy, Romania), Mihaela-Irma VLADESCU, Alina RĂDOII and Ionut GÂF DEAC (Centre for Demographic Research "Vladimir Trebici", Romania)

*Abstract:* Even if, according to studies, a characteristic of women's migration is the fact that they generally leave individually and not in groups, this type of external displacement produces multiple consequences, both at a macroeconomic and microeconomic level. At the level of the European Union countries, the number of emigrants differs from one country to another, this being influenced by the degree of development of the destination country and not least by the opportunities offered to immigrants. Social, economic and environmental differences between the countries of origin and the destination countries influence the migration process and simultaneously create a number of opportunities and obstacles for the migrant. In the study, we identified with the Pearson correlation coefficient the influence of the poverty rate on the number of migrant women. In countries such as Romania and Bulgaria, where the poverty rate among women is higher, the number of female emigrants is higher than that of immigrants, deepening the labor force deficit, among other things, while in more developed countries, the increase immigration is positive, contributing to the increase in the population of the country of adoption. Although over time the reasons why women choose to migrate have diversified, in less developed countries, poverty and social exclusion still represent a predominant cause of this phenomenon.

**Adina OPRIȘAN** (New Mexico State University, USA)

*Limit theorems for functionals of semi-Markov processes*

*Abstract:* In this talk we focus on almost sure central limit theorems for some classes of ergodic Markov processes, including autoregressive and Markov renewal processes, and for ergodic semi-Markov processes. The sequence of empirical processes defined as logarithmic averages of the additive functionals of semi-Markov processes satisfies an almost sure weak convergence and a large deviation principle. The rate function is given as a specific relative entropy of the Wiener measure on Skorokhod space.

**Christina PARPOULA** (Panteion Univ. of Social and Political Sciences, Greece)

*Change-point analysis for Public Health surveillance and decision-making*

joint work with Alex KARAGRIGORIOU (Univ. of the Aegean, Greece)

*Abstract:* Surveillance is a core public health activity that provides information vital for the protection and promotion of health. In particular, surveillance is critical for detecting disease outbreaks rapidly and for guiding interventions to effectively control epidemics. Considerable research has been directed towards early detection of the start of the epidemic, in order to initiate a timely response, and rarely the focus has been given on the whole signal and/or the end of the epidemic. Toward this end, this study aims at evaluating the ability of change-point analysis methods to identify the full-time course of an epidemic and locate the whole disease outbreak signal. Depending on the underlying model used to solve the change-point problem, we compare the performance of some state of the art parametric, nonparametric and Bayesian change-point model approaches with those considered as “gold standard” methods. The empirical and simulation-based results support that change-point analysis is a useful analytic tool that can be extensively used to understand disease development, evaluate the design of new strategies of prevention and control of the disease, and thus steer public health decision-making processes.

*References*

- [1] Parpoula, C. (2022). A distribution-free control charting technique based on change-point analysis for detection of epidemics. *Statistical Methods in Medical Research*, 31(6), 1067-1084.
- [2] Parpoula, C., A. Karagrigoriou. (2020). On Change-Point Analysis-Based Distribution-Free Control Charts with Phase I Application. In Koutras M., Triantafyllou I., eds. *Distribution-free Methods for Statistical Process Monitoring and Control*, Springer International Publishing, 157-182.

**Florin Marius PAVELESCU** (Institute of National Economy, Romania)

*A proposal for the generalization of Onicescu Informational Energy*

*Abstract:* Informational Energy(IE) defined by O. Onicescu (1966) has many applications indifferent fields of technical activities. The respective concept was developed by A. Theodorescu(1977), L. Pardo (1986) J.A Prado, M. Vicente (1994) and M.C. Pardo, J.A. Pardo (1995).Therefore, there are proposed new concepts such as informational correlation, weighted informational energy or useful informational energy. In this paper, the author proposes a generalization of IE, called generalized standardized informational energy (GStIE) and the identification of the correlation between IE or GStIE and other indicators such as standardized informational energy, the useful informational energy and generalized weighted variance.

**Eugen PIRCALABELU** (UC Louvain, Institute of Statistics, Biostatistics and Actuarial Sciences, LIDAM, Belgium)

*Linear manifold modelling and graph estimation based on multivariate functional data with different coarseness scales*

joint work with Gerda CLAESKENS (KU Leuven, ORSTAT and Leuven Statistics Research Center, Belgium)

*Abstract:* We develop a high-dimensional graphical modelling approach for functional data where the number of functions exceeds the available sample size. This is accomplished by proposing a sparse estimator for a concentration matrix when identifying linear manifolds and by making use of regularized procedures that estimate sparse undirected graphical models. By doing so, one also gets insight into the conditional independence structure that governs the multivariate functional data, while at the same time estimating linear combinations of the components. By working in a penalized setting our contribution enriches the functional data framework by estimating sparse undirected graphs that show how functional nodes connect to other functional nodes. As such, the proposed procedure extends the ideas of the manifold representation for functional data to high-dimensional settings where the number of functions is larger than the sample size. It allows multiple coarseness scales to be present in the data and proposes a simultaneous estimation of several related graphs, that are constrained to share similarities due to the design of the problem. Simulated and real data examples show beneficial performance pointing to the procedure being a useful tool for modeling multiscale, multivariate functional data.

**Ionel POPESCU** (Univ. of Bucharest & IMAR, Romania)

*Fragility of limiting distributions for a learning model*

joint work with Tushar Vaidya (Nanyang Technological Univ. Singapore, Singapore)

*Abstract:* I will introduce a model for interaction and learning in financial markets and study the effect of noise addition to it. In some cases we actually obtain some limiting distributions which seem to be very sensitive to the noise and in rare cases we obtain CLT.

**Mihaela PRICOP-JECKSTADT** (Univ. Politehnica of Bucarest, Romania)

*Functional data analysis with indirect observations*

*Abstract:* Optimal mean estimation from noisy independent paths of a stochastic process that are indirectly observed is an issue of great interest in functional inverse problems. In this paper, minimax rates of convergence for a class of linear inverse problems with correlated noise, general source conditions and various degrees of ill-posedness are proven in a continuous setting, when the paths are entirely observed, and in a projected framework. The phase transition phenomenon characteristic to the functional data analysis appears also here and the thresholds that separate the sparse and the dense data set scenarios are computed for different smoothness conditions. The common design proves to be a special case of the independent design in view of the interpretation of the sampling properties via s-numbers and the price to pay for the data correlation turns out to be high. Finally, the theoretical results are illustrated in various settings for a classical linear inverse problem.

*References*

- [1] Pricop-Jeckstadt M. Optimal indirect estimation for linear functional inverse problems. Submitted to *Inverse Problems*.
- [2] Mathe, P. and Pereverzev, S., (2006). Regularization of some linear ill-posed problems with discretized random noisy data. *Math. Comp.*, (75).
- [3] Zhang, X. and Wang, J.L., (2016). From sparse to dense functional data and beyond. *The Annals of Statistics*, (44).

**Marius RĂDULESCU** (ISMMA, Romania)

*Limit of powers of matrices with applications to Markov chains*

joint work with Sorin RĂDULESCU (Univ. of Craiova, Romania)

*Abstract:* Several conditions that imply the convergence of powers of arbitrary square matrices or of sequences of matrices that are built from powers of matrices to a square matrix are given. The description of the limiting matrix is also presented. Applications to finite Markov chains are discussed also.

**Adrian - Radu REY** (School for Advanced Studies of the Romanian Academy & Mountain Economy Centre "CE-MONT" & INCE, Romania)

*Aspirations and priorities of young people for the future of mountain areas in Europe*

joint work with Laura Giorgiana ANGHEL (School of Advanced Studies of the Romanian Academy, Romania), Luminita CHIVU (INCE, Romania)

*Abstract:* Like all young people everywhere in Europe, those living in the mountains also dream of modernity, openness, travel, exchanges and opportunities for professional and economic development. They all want to be connected to the world, to invest in a profession or business they like. Even if traditional occupations still continue to attract numerous young people, many also aspire to work in industry or service sectors. However, most young people are increasingly concerned about the difficulty of finding a job or investing in a business that will provide them with a decent income and a prosperous future in the turbulent times we are going through and rightly so if we are looking at unemployment figures in different European countries. But their professional future is not their only concern. Quality of life, the opportunity to doing sports, access to cultural activities and especially the quality of the relationships they maintain within the communities to which they belong, occupy a preponderant place in their life project. Most young people living in the mountains want to stay and work there, and the results of the latest surveys show that Romania ranks first in Europe. And that's good news! However, in many cases, the inability to find in the mountains the services they need, the training and occupations they want, causes them to migrate to larger cities in Romania where there are universities and where better economic opportunities are concentrated or to other more developed countries from which they do not always return, this phenomenon being almost irreversible and with a negative impact on the economy of the mountain areas. Can young people fulfill their aspirations by staying in the mountains or by leaving temporarily for studies or relevant professional experience after which they can return and develop their life projects, enriched by their skills and experience? What are the sectors that attract them? What are the obstacles that keep those who want to settle in the mountains or those who want to stay there? These are all key questions to which science and public policies must provide answers and solutions. Maintaining and attracting young people to mountain areas is a strategic issue of utmost importance and with long-term effects on the mountain economy and sustainable development of Romania and of many other mountainous countries in Europe.

**Steliana RODINO** (Institute of Research for Agricultural Economics and Rural Development, National Institute of Research and Development for Biological Sciences)

*A system dynamics approach for shared socio-economic pathways on Danube mouths region*

joint work with Ruxandra POP (Institute of Research for Agricultural Economics and

Rural Development, Romania), Luminita LAZAR (National Institute for Marine Research and Development "Grigore Antipa", Romania)

**Sanda ROȘCA** (Department of Physical and Technical Geography, Faculty of Geography, Babes Bolyai Univ., Romania)

*Delphi algorithm approach for multirisk assessment in protected natural areas from Romania*

joint work with Sorin AVRAM (INCA, Department of Geography, University of Craiova, Romania), Carmen GHEORGHE (INCE, Romania)

**Grigore Teodor SAMOILĂ** (School of Advanced Studies of the Romanian Academy, Romania)

*Testing the dependence of the average value of household expenditure on culture on the level of final household consumption, government expenditure and newly created value in the culture sector*

joint work with Carmen GHEORGHE (INCE, Romania)

*Abstract:* Since for the analysis of the specific expenses of cultural services we did not identify in the specialized literature models in which the consumption of cultural services is explained by the gross added value in the field of art, performances and recreation, we proposed the modeling of the relationship between the evolution of the indicator "expenditures of households for services recreation and culture" and the evolution of indicators: general government spending on cultural services, gross added value in the field of art, performances and recreation and the average net monthly salary (euro) we chose the VAR (Vector Auto regression) model. Vector Auto regression type model, it allows the symmetrical treatment of all the variables in the model, in the sense that it does not implicitly assume the exogeneity of a certain variable (as happens in the case of the transfer function analysis), thus analyzing the existence of a dynamic dependency relationships between the evolution of the household expenditure indicator for recreation and culture services and the evolution of the indicators: general government expenditure on cultural services, gross added value in the field of art, performances and recreation; other activities of extra-territorial households, organizations and bodies and average monthly net salary earnings.

**Răzvan-Cornel SFETCU** (Univ. of Bucharest, Romania)

*Divergences between probability measures*

joint work with Sorina-Cezarina SFETCU (Univ. of Bucharest, Romania) and Vasile PREDA (ISMMA & Nat. Inst. for Ec. Res. INCE & Univ. of Bucharest, Romania)

*Abstract:* We prove the pseudo-additivity, non-negativity, monotonicity and find some bounds for Tsallis and Kaniadakis divergences between two probability measures.

**Florentina SUTER** (Univ. of Bucharest & ISMMA, Romania)

*Dynamic information measures for the concomitants of order statistics*

*Abstract:* In this presentation we will discuss some dynamic information measures for the concomitants of order statistics from bivariate families with exponential margins.

**Florentin ȘERBAN** (Bucharest Univ. of Economic Studies & Univ. of Bucharest, Romania)

*Optimal reinsurance models in a dynamic framework*

*Abstract:* The aim of this paper consists in deriving the optimal reinsurance strategy from an insurer's perspective. The main objective is to construct the optimal policy which maximizes the insurer's survival probability. Closed formulas for the optimal reinsurance strategies and their corresponding survival probabilities under proportional and excess-of-loss reinsurance are derived, by using dynamic programming techniques.

**Aida TOMA** (Bucharest Univ. of Economic Studies & ISMMA, Romania)

*Robust estimators for moment condition models*

joint work with Amor KEZIOU (Univ. of Reims, France), Luiza BĂDIN (Bucharest Univ. of Economic Studies & ISMMA, Romania) and Silvia DEDU (Bucharest Univ. of Economic Studies, Romania)

*Abstract:* We present robust minimum empirical divergence estimators for moment condition models, based on truncated orthogonality functions and dual forms of divergences. We discuss asymptotic properties for these estimators, robustness properties, as well as their equivariance in the case when the model is invariant with respect to additive or multiplicative transformations groups. Some examples based on Monte Carlo simulations illustrate the performance of the estimation method. (This work was supported by a grant of the Ministry of Research, Innovation and Digitization, CNCS/CCCDI-UEFISCDI, project number PN-III-P4-ID-PCE-2020-1112, within PNCDI III).

**Ciprian TUDOR** (Univ. of Lille, France)

*Exact variation and drift parameter estimation for the nonlinear fractional stochastic heat equation*

joint work with Julie GAMAIN (Univ. of Lille, France)

*Abstract:* We consider the fractional stochastic heat equation driven by a nonlinear Gaussian space-time white noise and we analyze its mild solution. We actually study the limit behavior of the spatial quadratic variation of its mild solution both in the linear and nonlinear noise cases by obtaining the exact limit of this quadratic variation. We apply these results to parameter estimation. More precisely, we construct an estimator for the drift parameter of the fractional stochastic heat equation with nonlinear noise, which is defined in terms of the quadratic variation and it is based on the observation of the solution at a fixed time and at discrete points in space. The proofs are based on the relation between the solution to the linear fractional stochastic heat equation and the fractional Brownian motion and on a sharp analysis of the Green kernel associated to the fractional Laplacian operator.

**Răzvan VASILE** (INCE & School of Advanced Studies of the Romanian Academy, Romania)

*Main determinants of the sustainable digital transformation of the labour force. Qualitative assessment based on a scientometric analysis*

**Valentina VASILE** (Institute of National Economy, Romania)

*Demographic differences in self-review at the Romanian Population Census. Statistical analysis of preliminary results*

joint work with Raluca MAZILESCU (Institute of National Economy, Romania), Ana-Maria CIUHU (Institute of National Economy, National Institute of Statistics, Romania)

**Gheorghiță ZBĂGANU** (ISMMA, Romania)

*Making tops: what is the chance to get a leader?*

joint work with Anișoara Maria Răducan (ISMMA, Romania)

*Abstract:* In the  $d$ -dimensional space the order is not total. However in many cases people want to know "who is the best " in the case of a population of  $n$  individuals in order to elect him in some function—for instance to be their king. Suppose that there exists an imponderable unidimensional characteristic  $X$  called "intrinsic value" which is not measurable. What can we do is to measure some effects of it  $f_j(X)$ ,  $j = 1, 2, \dots, d$ . and to



compare the vectors  $\mathbf{Z} = \mathbf{f}(X)$  measured for all individuals. Here  $\mathbf{f} = (f_1, \dots, f_d)$ ,  $d > 1$ . Our question is: what is the probability that a leader does exist among  $n$  vectors obtained in that way. Give a sense to this naive question. Answer to it at least in the simplest probability scenarios. If not exactly, at least asymptotically.